SciML Journal Club Meeting 2/26

On “FLOWER: A Flow-Matching Solver for Inverse Problems”

Presentation by Zac

Paper Details

This is the paper btw!!

Authors: Pourya, et al.

- Great Swiss research school, focused in biomedical imaging

Topic:

- Using Flow Matching for Inverse Problem solving in Image Tasks

Reception:

- Accepted at ICLR this year
- Not a ton of traction yet
- Final manuscript not out yet

Published as a conference paper at ICLR 2026

Flower: A FLOW-MATCHING SOLVER FOR INVERSE PROBLEMS

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ABSTRACT

We introduce *Flower*, a solver for linear inverse problems. It leverages a pre-trained flow model to produce reconstructions that are consistent with the observed measurements. *Flower* operates through an iterative procedure over three steps: (i) a flow-consistent destination estimation, where the velocity network predicts a denoised target; (ii) a refinement step that projects the estimated destination onto a feasible set defined by the forward operator; and (iii) a time-progression step that re-projects the refined destination along the flow trajectory. We provide a theoretical analysis that demonstrates how *Flower* approximates Bayesian posterior sampling, thereby unifying perspectives from plug-and-play methods and generative inverse solvers. On the practical side, *Flower* achieves state-of-the-art reconstruction quality while using nearly identical hyperparameters across various linear inverse problems. Our code is available at https://github.com/mehrsapo/Flower.

1 INTRODUCTION

Inverse problems are central to computational imaging and computer vision (McCann & Unser, 2019; Zeng, 2001). Their goal is to reconstruct an underlying signal $\mathbf{x} \in \mathbb{R}^d$ from its observed measurements $\mathbf{y} \in \mathbb{R}^M$. Here, we focus on linear inverse problems, such that the acquisition of the measurements follows the model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

for some linear forward operator $\mathbf{H}\colon \mathbb{R}^d \to \mathbb{R}^M$ and additive white Gaussian noise $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_n^2\mathbf{I})$. From a Bayesian perspective, the simplest reconstruction approach is to obtain the maximum-likelihood estimation

$$\hat{\mathbf{x}}_{\mathrm{MLE}} = \arg\max_{\mathbf{x}\in\mathbb{R}^d} p_{\mathbf{Y}|\mathbf{X}=\mathbf{x}}(\mathbf{y}) = \arg\min_{\mathbf{x}\in\mathbb{R}^d} \frac{1}{2\sigma_n^2}\|\mathbf{H}\mathbf{x}-\mathbf{y}\|_2^2. \quad (2)$$

However, this problem is ill-posed and yields poor-quality solutions. Another approach is to obtain the maximum a posteriori estimation (MAP)

$$\hat{\mathbf{x}}_{\mathrm{MAP}} = \arg\max_{\mathbf{x}\in\mathbb{R}^d} p_{\mathbf{X}|\mathbf{Y}=\mathbf{y}}(\mathbf{x}) = \arg\min_{\mathbf{x}\in\mathbb{R}^d} \left(\frac{1}{2\sigma_n^2}\|\mathbf{H}\mathbf{x}-\mathbf{y}\|_2^2 - \log p_{\mathbf{X}}(\mathbf{x})\right), \quad (3)$$

which requires the knowledge of the prior distribution $p_{\mathbf{X}}$ of images, a quantity that is generally unknown. The minimization problem of equation 3 is consistent with the variational perspective

Agenda

1. Overview of Problem Space and Proposed Contribution
2. Background on Formulating Inverse Problem
3. Explanation of FLOWER
4. Results on Toy and Real* Tasks
5. Discussion on Paper and Applications

* If AFHQ-Cat and CelebA are considered real….

A (Very) Brief Overview

Current State:

- Interest in Plug-and-Play (PnP) Diffusion Priors for Inverse Problems in imaging and other domains is growing.
- Flow Approaches exist, but with little theoretical support*
- Flow Models have shown to be effective and efficient compared to diffusion, so solvers could be faster and more scalable

Contributions:

- FLOWER: FLOW-Based inverse problem solvER
- Bayesian Analysis and relation to other PnP literature
- Numerical Validation on Toy and Realistic Datasets on variety of inverse tasks

Background: Inverse Problems in Image Tasks

Forward Problem (FP):

- Taking a system or domain and solving for a final state based on a known model.
- An example of a model are partial differential equations (PDEs) for diffusion, advection, waves, .. etc
- Given an input **x,** a forward operation(s) F(**x**) exists to get an output **y**
- Normally “well posed” in contexts of interest.

Inverse Problem (IP):

- Taking observations of a state and predicting initial conditions or model parameters.
- Taking observations of **y** and/or **x** and a general model F(x) and inferring info about any of them
- Normally “ill-posed” making solves computationally expensive and difficult

IP for Image Generation:

- Common examples are Deblurring, Infilling, superposition, ..., etc.
- This paper focuses on **linear** inverse problems so:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- Where x is the original image, **H** is some transformation or forward operator, and **n** is additive white noise $\mathbf{n} \sim \mathbf{N}(0, \sigma_n^2 \mathbf{I})$

Background: Proximal Operators

Proximal Operator:

- Define as:

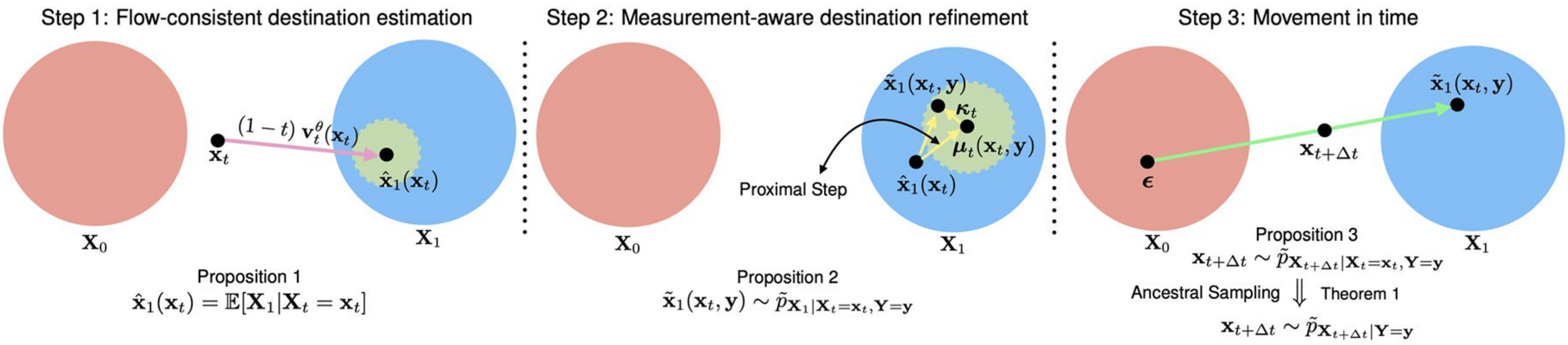
$$\text{prox}_y(x) = \text{argmin}_{w \in R^d}(½ \,||\, w - x \,||_2^2 + f(w))$$

- Meant to project x onto a set associated with f, balancing proximity to x and regularized by f.
- Will be used here to enforce data consistency between generated images and observations

FLOWER: Overview

High Level Procedure:

1. Flow-consistent destination estimation
2. Measurement-aware destination refinement
3. Movement in Time



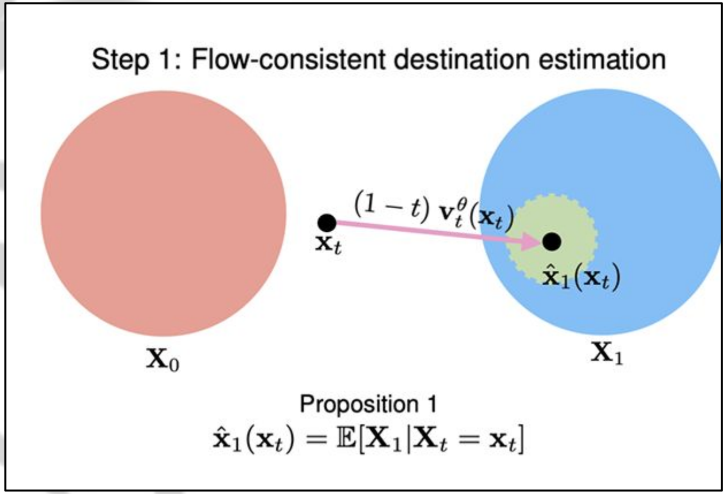
FLOWER: Destination Estimation (1)

Flow Matching to the End:

- At time t, use the pretrained flow model $v_t^\Theta(x_t)$ to estimate the final image x_1

$$\hat{\mathbf{x}}_1(\mathbf{x}_t) = \mathbf{x}_t + (1-t)\mathbf{v}_t^\theta(\mathbf{x}_t).$$

- This amounts to taking a normal flow update, but scaled by 1-t instead of dt
- This will be a 1 step flow update



FLOWER: Measurement Aware Refinement (2)

Refine the Current Estimate and Add Noise:

Get an estimate close to the observations:

$$\tilde{\mathbf{x}}_1(\mathbf{x}_t, \mathbf{y}) = \boldsymbol{\mu}_t(\mathbf{x}_t, \mathbf{y}) + \gamma \boldsymbol{\kappa}_t$$

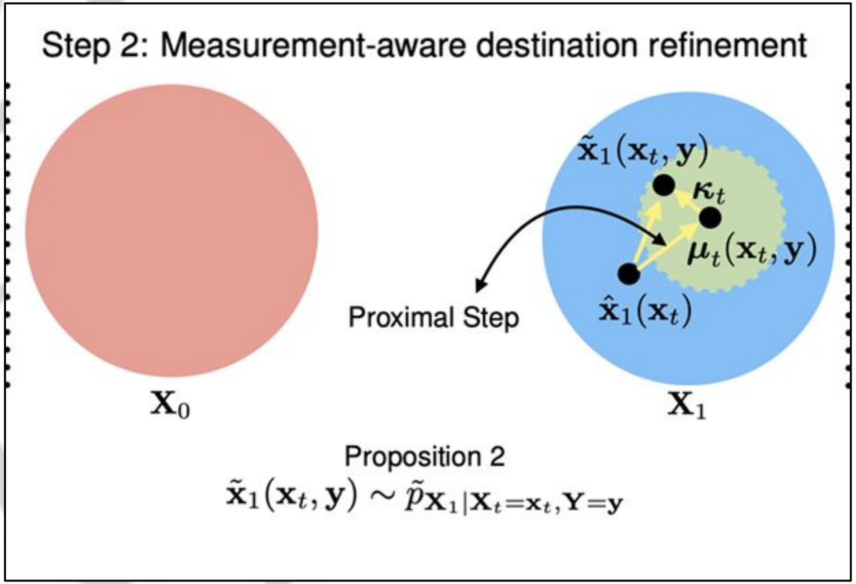
where

$$\boldsymbol{\mu}_t(\mathbf{x}_t, \mathbf{y}) = \operatorname{prox}_{\nu_t^2 F_{\mathbf{y}}}\left(\hat{\mathbf{x}}_1(\mathbf{x}_t)\right), \quad \boldsymbol{\kappa}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_t),$$

and

$$F_{\mathbf{y}}(\mathbf{x}) = \frac{1}{2\sigma_n^2}\left\|\mathbf{H}\mathbf{x} - \mathbf{y}\right\|_2^2$$

$$\nu_t = \frac{(1-t)}{\sqrt{t^2 + (1-t)^2}} \qquad \boldsymbol{\Sigma}_t = \left(\nu_t^{-2}\mathbf{I} + \sigma_n^{-2}\mathbf{H}^\top\mathbf{H}\right)^{-1}$$



FLOWER: Movement in Time (3)

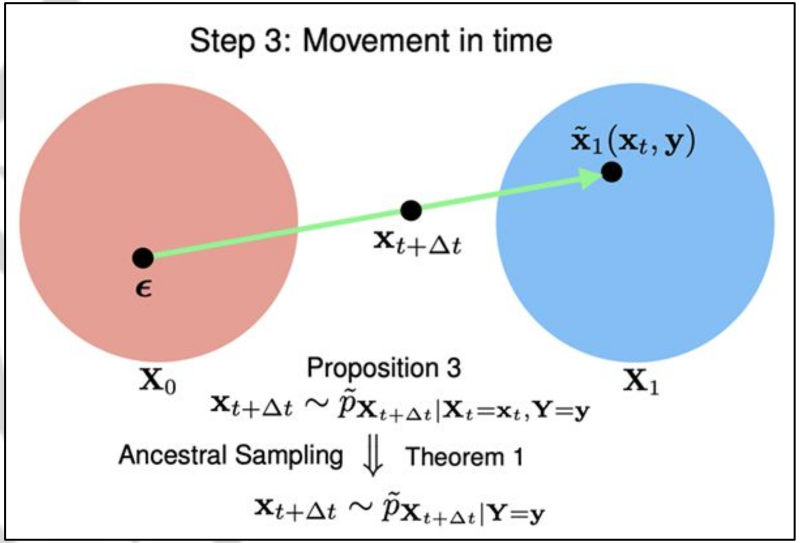
Section 3:

Back Project to time t = t + delta

- From the refined estimate x_1 tilde, project back onto the flow path with:

$$\mathbf{x}_{t+\Delta t} = (1 - t - \Delta t)\epsilon + (t + \Delta t)\tilde{\mathbf{x}}_1(\mathbf{x}_t, \mathbf{y}),$$

- Add the epsilon term (a new sample from X_0 to inject some noise into this step.)
- Repeat 1 - 3 at each iteration

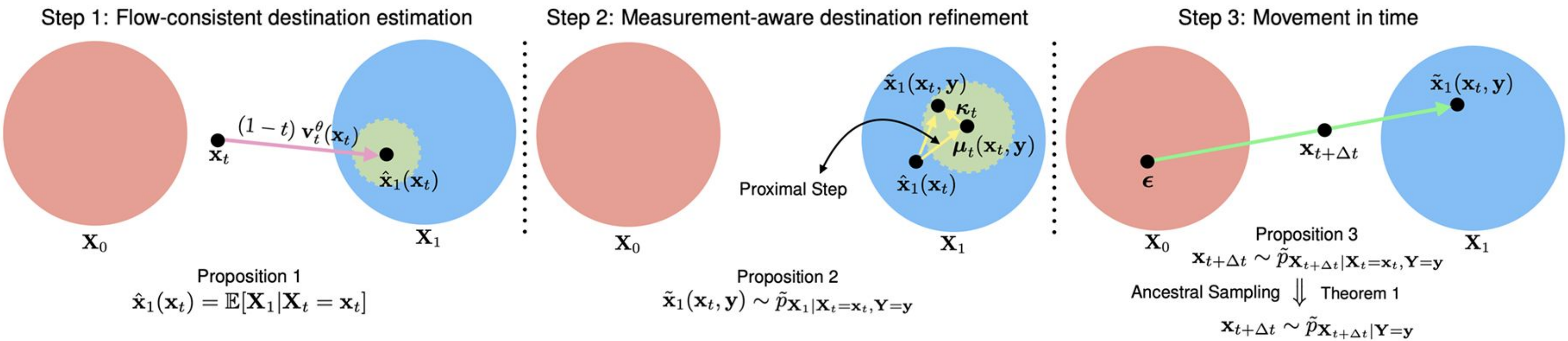


FLOWER: Recap

Section 3:

At Each Iteration from 1 to N do:

1. Flow-consistent destination estimation
2. Measurement-aware destination refinement
3. Movement in Time



FLOWER: A Note on the Math….

Section 3:

Relation to Bayesian Inference and Ancestral Sampling:

- We are not estimating the score of the posterior p(**x|y**) like a DPS approach
- Instead we approximate $p_{xt + dt \mid xt,\, y}(\mathbf{x}_{\mathbf{t + dt}})$.
- We use this to build $p_{x1 \mid xt,\, y}(\mathbf{x}_{\mathbf{1}})$ which will be a gaussian approximating the true posterior using the chain rule.

Notes on Assumptions and what you end up estimating

- $\mathbf{X}_{\mathbf{0}}$ and $\mathbf{X}_{\mathbf{1}}$ distributions must be independent, with no OT coupling
- The forward operator **H** is linear

Experiments:

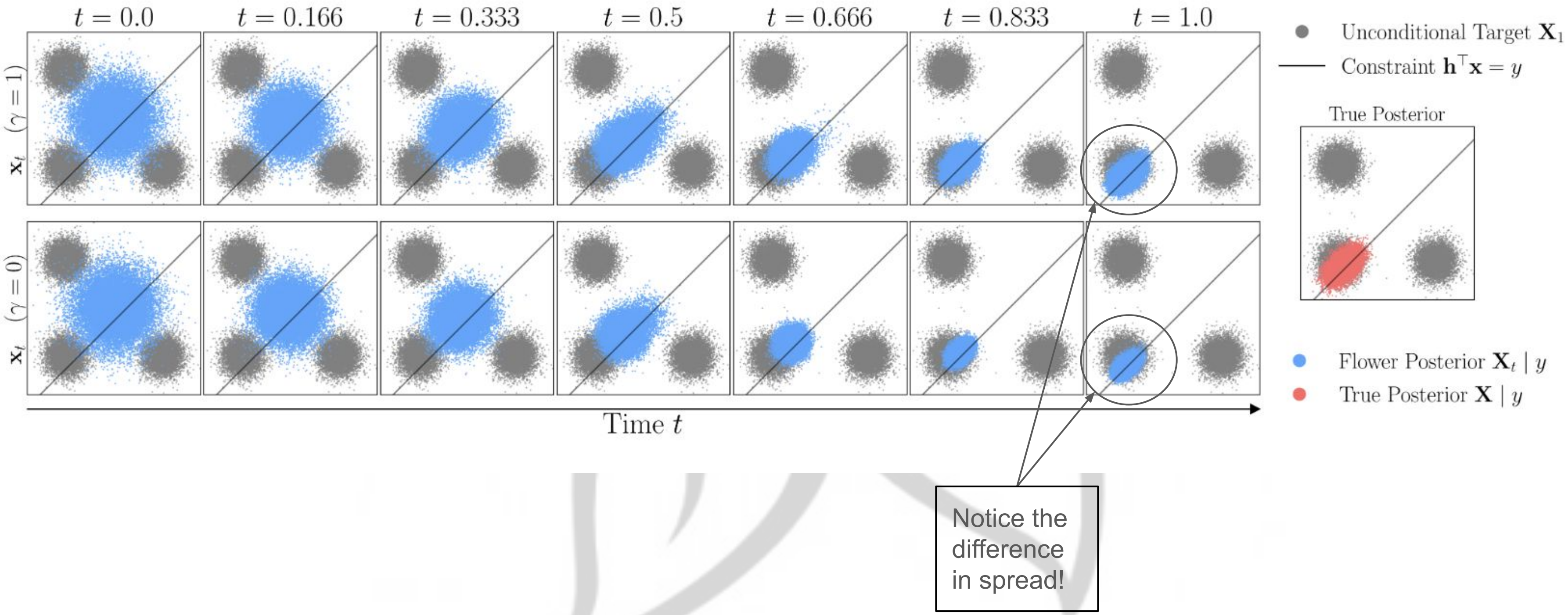
Toy Experiment:

- Validate on a Gaussian Mixture Model where true posterior is available
- Show that FLOWER will approximate the posterior

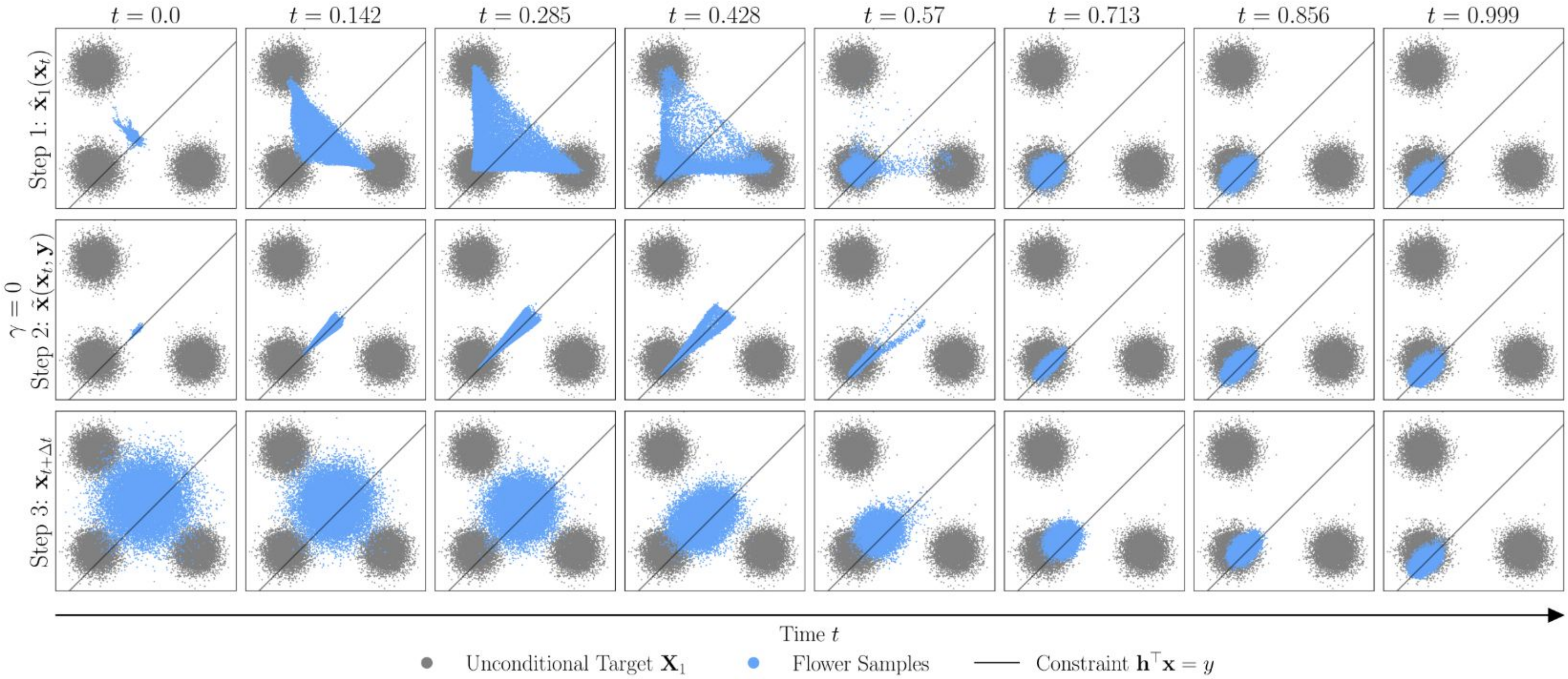
Real Image Experiments:

- Class image inverse problem tasks
  - Box Inpainting, Random Inpainting, Super-Resolution, Deblurring and Denoising
- CelebA and AFHQ-Cat datasets
- Compare against other flow models

Experiment 1: Mixture of Gaussians



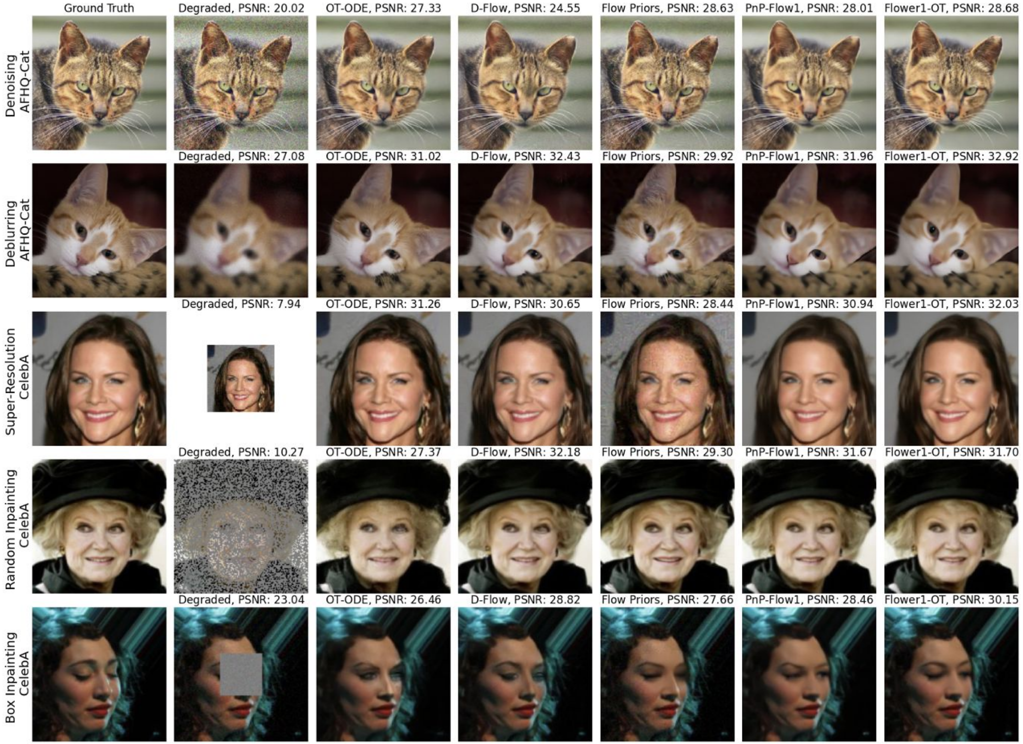
Experiment 1: Mixture of Gaussians (cont)



Experiment 2: Inverse Problems AFHQ-Cat + CelebA

Takeaways:

- Full table of numerical results in paper
- Flower performs well but not clear its the best for any one metric
    - LPIPS, PSNR, SSIM
- With some massaging and choice in reporting, they find it with OT to be the best for most tasks



Experiment 2: Inverse Problems AFHQ-Cat + CelebA (cont)

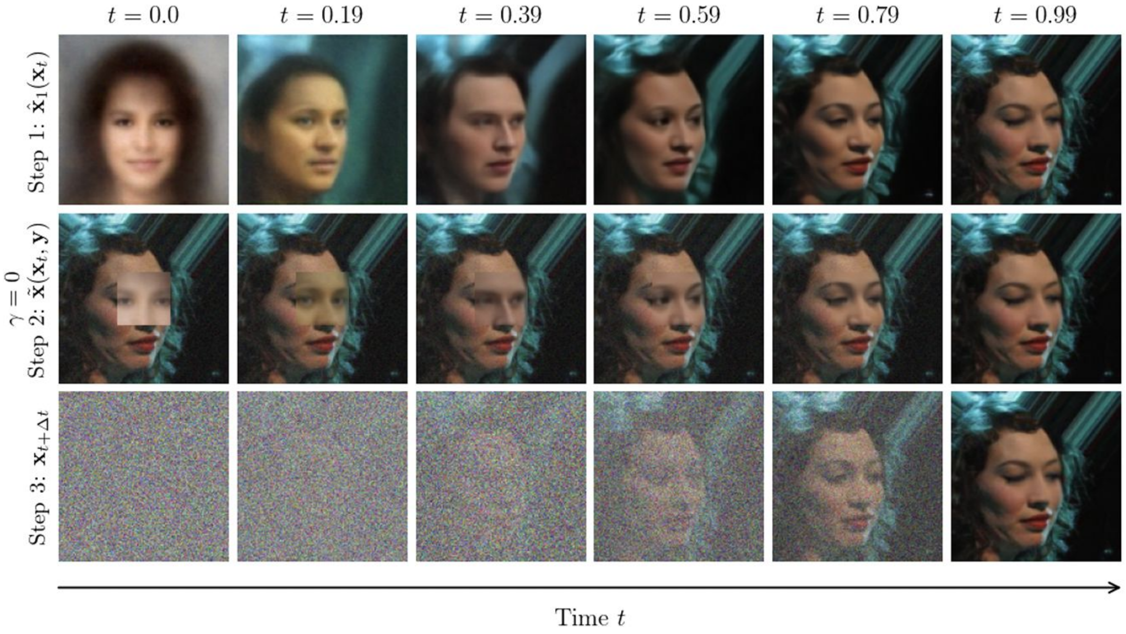


Figure 4: Solution path of *Flower* for box inpainting.

Final Takeaways and Discussion

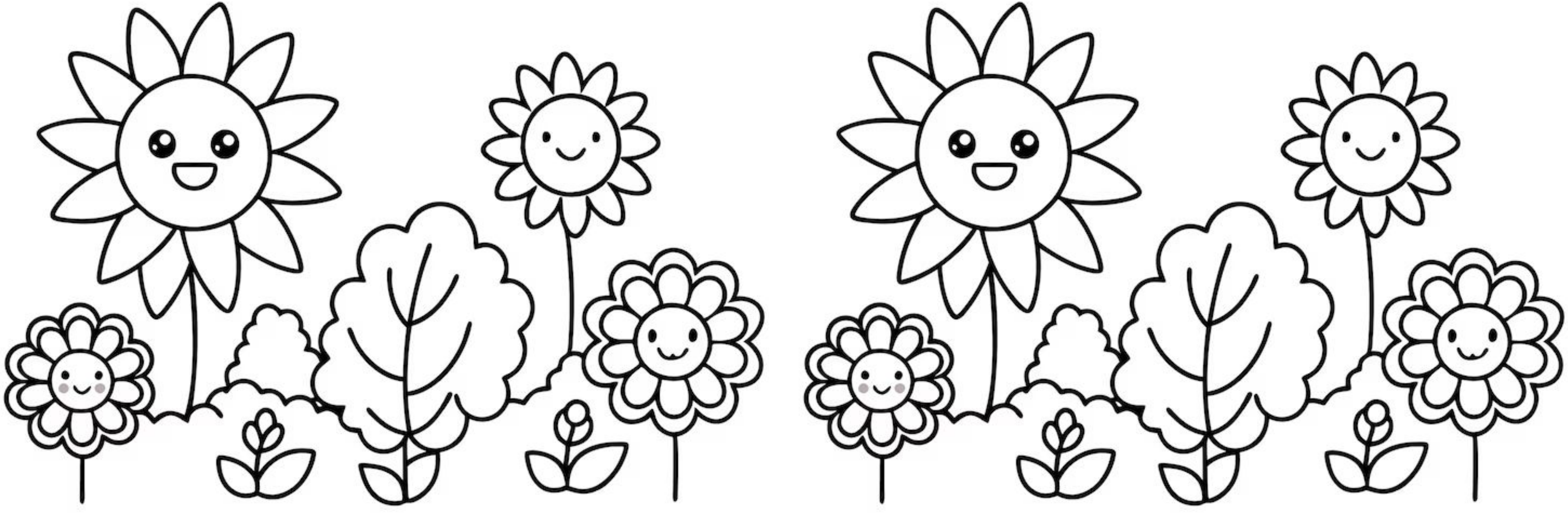
Last Thoughts on the Paper:

- Pros:
  - Introduces a nice theoretical backing for why flow models can approximate posteriors
  - Cements Flow Matching's place in the PnP literature
  - Show good results compared to other flow models
- Cons:
  - Unrealistic framing of their contribution (other models + sampling strategies exist)
  - Scope of theory is limited to independent couplings, approximating gaussians,... etc
  - Metrics and use case are not well justified in their difference from previous papers

Can we apply this in our works?

- Yes!
  - PDE Inverse problems following the same posterior sampling procedure
  - FLOWER (and other samplers) go on top of flow models, just affecting inference

Thanks!



SciML Club Official Business

Confirm Speakers for Weeks 9 and 10 and beginning of Spring term:

- Week 9: Manish
- Week 10:
- Week 1 Sping Term:
- Week 2 Spring Term:

No Meeting Finals Week!

Time and Place for Spring ‘26 meeting TBD

- Will send out another Whenisgood poll during spring break and reminder in week 1
- Tentatively plan on same time as now for week 1

Curating a Reading List

- Goal is to build a list of good papers in the area of SciML
    - (Not all of our past papers have been good, so pruning that list too :/)
- Want everyone’s help!
    - If you have read or read good papers in the future please let us know!
    - Joe is our website guru
    - A good list will also help folks pick papers if you’re in a rush :)
    - Think seminar list of papers, but they actually are good lol